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Professor Sohrab KORDROSTAMI, PhD
E-mail: sohrabkordrostami@gmail.com
Department of Mathematics
Lahijan Branch, Islamic Azad University, Lahijan, Iran
Monireh JAHANI SAYYAD NOVEIRI, PhD (Corresponding author)
E-mail: monirehjahani@yahoo.com
Department of Mathematics
Lahijan Branch, Islamic Azad University, Lahijan, Iran

COST EFFICIENCY ANALYSIS OF HETEROGENEOUS NETWORK PROCESSES

Abstract. *Cost efficiency in network systems has been addressed in the existing data envelopment analysis (DEA) approaches considering homogeneous technology. Due to the presence of the heterogeneity of production technologies in different network processes, this paper attempts to develop the meta-frontier network DEA models to analyze convex and non-convex meta-frontier cost efficiencies. Technological gaps can influence the overall production frontier due to technological gaps of the stages one and two. The group cost efficiency and meta cost efficiency are assessed to establish cost gap ratios of systems. The proposed approach is clarified using an application of soft drinks companies presented in the literature. The results show the usefulness of the provided approach in this paper to assess the cost efficiency of heterogeneous network processes and to estimate cost gap ratios and meta cost inefficiency sources.*

Keywords*: Data envelopment analysis, Cost efficiency, Convex meta-frontier, Non-convex meta-frontier, Network.*

JEL Classification: C06, C61, C67

1. Introduction

The cost management and the cost efficiency analysis are including significant issues for managers and decision makers in today's competitive world. One of beneficial tools to evaluate the cost efficiency of decision making units (DMUs) is the non-parametric data envelopment analysis (DEA) approach that originally was raised by Charnes et al. (1978). In the existing DEA studies, different models can be found to assess the performance of entities in distinct

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conditions such as containing undesirable factors (Halkos and Petrou, 2019), flexible measures (Cook and Zhu, 2007), and so forth. The matters of cost efficiency and allocative efficiency were initially presented by Farrell (1957). Later, Fare et al. (1985) expanded them using linear programming methods. Tone (2002) provided an alternative approach to evaluate the cost efficiency so as to tackle the shortcomings of the existing methods until then. Furthermore, because of the presence of complex network structures in many real world applications, there are extensive studies on this issue in the DEA literature; see (Chen et al., 2009; Jahani Sayyad Noveiri et al., 2017; Jahani Sayyad Noveiri et al., 2019; Li et al., 2012) for instance. Lozano (2011) developed a network DEA approach to evaluate technical, scale, cost and allocative efficiency values of homogeneous network processes. Also, Banihashem et al. (2013) determined the cost, revenue and profit efficiencies of analogous multi-stage supply chains by using approaches based on network DEA.

In the DEA literature, there are also some approaches to compare DMUs across different technologies. Actually, entities may be members of disparate groups due to various environmental characteristics of them. O'Donnell et al. (2008) appraised metafrontiers and group frontiers by using DEA and stochastic frontier analysis (SFA) methods. Huang et al. (2015) described the meta Malmquist productivity index and the meta cost Malmquist productivity index using the DEA method under constant returns to scale and also the productivity gap was measured as the ratio of the group-specific productivity index and the meta productivity index. The given approach, moreover, used to examine Taiwanese and Chinese banks. Cho (2018) further provided the cost metafrontier Malmquist productivity index through the variable returns to scale assumption and determined the cost scale efficiency change and the sources of inefficiency. To illustrate in more details, DEA-based approaches under variable returns to scale were presented to evaluate the group cost efficiency and the meta cost efficiency of black-box structures under convex technology. Zhang et al. (2013) rendered a DEA approach based on the directional distance function to investigate technology gaps in fossil fuel electricity generation. Wang et al. (2013) used the metafrontier DEA approach to address the energy efficiency and applied it to estimate the energy efficiency of China's provinces. Sun et al. (2017) measured the performance of heterogeneous bank supply chains employing the directional distance function and metafrontier models. Chao et al. (2018) presented the convex metafrontier DEA model to calculate the profitability and marketability efficiencies and also to estimate technology gaps in heterogeneous Taiwanese banks. Yu and Chen (2020) introduced a metafrontier network DEA method to explore the technology biases in the each stage and to judge the favorite directions of the technological progress of entities. Majority of metafrontier DEA models are under the convex technology. However, some studies such as Afsharian and Podinovski (2018) dealt with non-convex metafrontier technologies. Afsharian and Podinovski (2018) presented an individual linear model to measure the efficiency of entities under the non-convex metatechnology and applied its dual to describe the returns to scale status of efficient entities on the metafrontier. However, as far we know, there is no systematic approach to examine the sources of the cost inefficiency and cost gap ratios in two-stage processes. Actually, preceding network DEA models to evaluate the cost efficiency such as they were presented in (Banihashem et al., 2013; Lozano, 2011) have considered different entities to be belong to an individual technology. Also, a few studies as in (Cho 2018; Huang et al., 2015) investigated the cost Malmquist productivity of DMUs imagined as black boxes under nonhomogeneous cost technology.

Therefore, the main intention of the present paper is to analyze cost efficiency of heterogeneous two-stage processes. To accomplish this purpose, network DEA frameworks are proposed to assess group cost efficiency values and meta cost efficiency rates of two-stage networks. Due to the presence of undesirable outputs in many real world situations, they are incorporated into the system under investigation. The weak disposability assumption of undesirable outputs is deemed to deal with them. Also, because of this fact that the metatechnologies may be convex or non-convex sets, two approaches are provided in this study to measure the cost efficiency under convex and non-convex metafrontiers. Cost gap ratios are determined for each component of two-stage networks. Moreover, the sources of meta cost inefficiency are addressed.

The rest of this paper is organized as follows: In section 2, two-stage DEA models are proposed to analyze the group cost efficiency and the meta cost efficiency under convex and non-convex metatechnolgies. Also, cost gap ratios and sources of meta cost inefficiencies are estimated for each of the stages. The application of soft drinks companies is provided to illustrate the proposed approach and indicate its reliability in Section 3. Finally, conclusions are given in Section 4.

2. The proposed technique

In this section, the group cost efficiency under convex technology and meta cost efficiencies regarding convex and non-convex technologies are analyzed in the system depicted in Figure 1. In this figure, $(\mathbf{x}^s, \mathbf{y}^s, \mathbf{b}^s, \mathbf{z}), s = 1, 2$ shows the vector of inputs, desirable outputs, undesirable outputs and intermediate measures for components 1 and 2. Two-stage systems may be managed under distinct technological levels. Thus, differences in technological level may happen in particular components. Nevertheless, biases of the group cost frontier and meta cost frontier can be disparate. Therefore, group cost efficiency, meta cost efficiencies under convex and non-convex technologies, cost gap ratios and the sources of meta cost inefficiency are addressed.

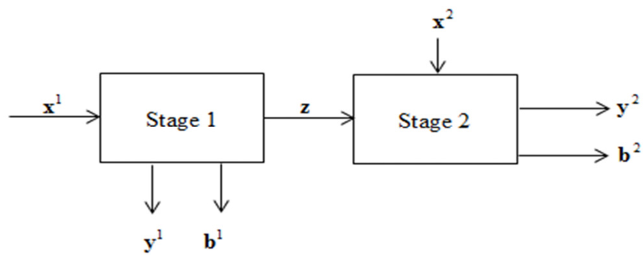

Figure 1 – A two-stage system

2.1. Group cost frontier analysis of network processes

Referred to Figure 1, suppose N two-stage processes as DMUs, $DMU_n (n=1,\ldots,N)$, are present. The inputs $x_{in}^1 (i \in I_1, n=1,\ldots,N)$ are used in stage 1 and desirable outputs $y_{rn}^1 (r \in R_1, n=1,\ldots,N)$, undesirable outputs $b_{un}^1 (u \in U_1, n=1,\ldots,N)$ and intermediate measures $z_{kn} (k \in K, n=1,\ldots,N)$ are produced. Intermediate measures $z_{kn} (k \in K, n=1,\ldots,N)$ and external inputs $x_{in}^2 (i \in I_2, n=1,\ldots,N)$ are also consumed in stage 2 and desirable outputs $y_{rn}^2 (r \in R_2, n=1,\ldots,N)$ and undesirable outputs $b_{un}^2 (u \in U_2, n=1,\ldots,N)$ are generated. The price values of inputs for stages 1 and 2 are denoted by $c_i^1, i \in I_1$ and $c_i^2, i \in I_2$, respectively. We assume $c_{in}^1 x_{in}^1 = \bar{x}_{in}^1, \forall i \in I_1, \forall n \in N$ and $c_{in}^2 x_{in}^2 = \bar{x}_{in}^2, \forall i \in I_2, \forall n \in N$. Due to the differences in production technologies, two-stage processes are classified into $N' > 1$ groups that $\sum_{n=1}^{N'} g^n = N, \Upsilon = \{1,\ldots,N'\}$. For g^{th} group, the first-stage cost-based production possibility set T_1^g and the second-stage cost-based production possibility set T_2^g can be shown as follows:

$$T_1^g = \{(\bar{\mathbf{x}}_g^1, \mathbf{y}_g^1, \mathbf{b}_g^1, \mathbf{z}_g) \mid \sum_{g=1}^{g^n} \lambda_g \bar{x}_{ig}^1 \le \bar{x}_i^1, \forall i \in I_1, \sum_{g=1}^{g^n} \lambda_g y_{rg}^1 \ge y_{ro}^1, \forall r \in R_1, \sum_{g=1}^{g^n} \lambda_g b_{ug}^1 = b_{uo}^1, \forall u \in U_1, \sum_{g=1}^{g^n} \lambda_g z_{kg} = z_{ko}, \forall k \in K,$$
$$\lambda_g \ge 0, g=1,\ldots,g^n\} \tag{1}$$

and

$$T_2^g=\{(\overline{\mathbf{x}}_g^2,\mathbf{y}_g^2,\mathbf{b}_g^2,\mathbf{z}_g)\,|\sum_{g=1}^{g^n}\mu_g\overline{x}_{ig}^2\le\overline{x}_i^2,\forall i\in I_2,\sum_{g=1}^{g^n}\mu_g y_{rg}^2\ge y_{ro}^2,\forall r\in R_2,\sum_{g=1}^{g^n}\mu_g b_{ug}^2=b_{uo}^2,\forall u\in U_2,\sum_{g=1}^{g^n}\mu_g z_{kg}=z_{ko},\forall k\in K,$$
$$\mu_g\ge 0,g=1,\dots,g^n\} \tag{2}$$

in which $\lambda_g(g=1,\dots,g^n)$ and $\mu_g(g=1,\dots,g^n)$ are intensity variables corresponding to stages 1 and 2 for the group g, respectively.

The network cost-based production possibility set can also be shown in the following way:

$$T_N^g=\{(\overline{\mathbf{x}}_g^1,\mathbf{y}_g^1,\mathbf{b}_g^1,\mathbf{z}_g,\overline{\mathbf{x}}_g^2,\mathbf{y}_g^2,\mathbf{b}_g^2)\,|\sum_{g=1}^{g^n}\lambda_g\overline{x}_{ig}^1\le\overline{x}_i^1,\forall i\in I_1,\sum_{g=1}^{g^n}\lambda_g y_{rg}^1\ge y_{ro}^1,\forall r\in R_1,\sum_{g=1}^{g^n}\lambda_g b_{ug}^1=b_{uo}^1,\forall u\in U_1,\sum_{g=1}^{g^n}\lambda_g z_{kg}=z_{ko},\forall k\in K,$$
$$\lambda_g\ge 0,g=1,\dots,g^n,\sum_{g=1}^{g^n}\mu_g\overline{x}_{ig}^2\le\overline{x}_i^2,\forall i\in I_2,\sum_{g=1}^{g^n}\mu_g y_{rg}^2\ge y_{ro}^2,\forall r\in R_2,\sum_{g=1}^{g^n}\mu_g b_{ug}^2=b_{uo}^2,\forall u\in U_2,\sum_{g=1}^{g^n}\mu_g z_{kg}=z_{ko},\forall k\in K,\mu_g\ge 0,g=1,\dots,g^n\} \tag{3}$$

By considering the aforementioned network cost-based production possibility set, the following model is provided to calculate the group cost efficiency of two-stage systems depicted in Figure 1:

$$E_N^{*g}=Min\ \frac{\sum_{i\in I_1}\overline{x}_i^1+\sum_{i\in I_2}\overline{x}_i^2}{\sum_{i\in I_1}c_i x_{io}^1+\sum_{i\in I_2}c_i x_{io}^2}$$

$$\begin{aligned}
s.t.\quad &\sum_{g=1}^{g^n}\lambda_g\overline{x}_{ig}^1\le\overline{x}_i^1,\forall i\in I_1, && (4.1)\\
&\sum_{g=1}^{g^n}\lambda_g y_{rg}^1\ge y_{ro}^1,\forall r\in R_1, && (4.2)\\
&\sum_{g=1}^{g^n}\lambda_g b_{ug}^1=b_{uo}^1,\forall u\in U_1, && (4.3)\\
&\sum_{g=1}^{g^n}\lambda_g z_{kg}=z_{ko},\forall k\in K, && (4.4)\\
&\sum_{g=1}^{g^n}\mu_g z_{kg}=z_{ko},\forall k\in K, && (4.5)\\
&\sum_{g=1}^{g^n}\mu_g\overline{x}_{ig}^2\le\overline{x}_i^2,\forall i\in I_2, && (4.6)\\
&\sum_{g=1}^{g^n}\mu_g y_{rg}^2\ge y_{ro}^2,\forall r\in R_2, && (4.7)
\end{aligned}$$

$$\sum_{g=1}^{g^n} \mu_g b_{ug}^2 = b_{uo}^2, \forall u \in U_2, (4.8)$$
$$\mu_g, \lambda_g \geq 0, \forall g. \tag{4}$$

Notice that constraints (4.4) and (4.5) have been taken in model (4) to incorporate the intermediate measures. We can also replace the constraints (4.4) and (4.5) with $\sum_{g=1}^{g^n} \lambda_g z_{kg} \geq \sum_{g=1}^{g^n} \mu_g z_{kg}, \forall k \in K$ to deal with free adjustable intermediate measures. Furthermore, weakly disposable undesirable outputs have been included in model (4) by taking constraints (4.3) and (4.8) into account. According to Fare et al. (Fare et al., 1989), outputs are weakly disposable if $(\mathbf{y}, \mathbf{b})$ belong to the production possibility set, then $(\theta \mathbf{y}, \theta \mathbf{b}), \forall 0 \leq \theta \leq 1$ is associated with the production possibility set.

In model (4), the overall two-stage process is called cost efficient for the unit under evaluation in g^{th} group if and only if $E_N^{*g} = 1$.

The unit under consideration, DMU_o, is said to be cost efficient in stage 1 and for g^{th} group if and only if $E_1^{*g} = (\sum_{i \in I_1} \bar{x}_i^{*1} / \sum_{i \in I_1} c_i x_{io}^1) = 1$. In the similar way, DMU_o is cost efficient in stage 2 and for g^{th} group if and only if $E_2^{*g} = (\sum_{i \in I_2} \bar{x}_i^2 / \sum_{i \in I_2} c_i x_{io}^2) = 1$.

In this subsection, the group cost efficiency of two-stage network systems was elaborated. In the next subsection, meta cost frontier efficiencies are dealt with for two-stage network designs.

2.2. Meta cost frontier analysis of network processes

In model (4), DMUs evaluated in special groups have similar production technologies. By regarding the metatechnology for the set of the group technologies as $T_N^M = T_N^1 \cup T_N^2 \cup ... \cup T_N^{N'}$, the meta technology can be convex or non-convex. In the following subsections, approaches are proposed to analyze the convex and non-convex meta-frontier cost efficiencies of two-stage processes.

2.2.1. Meta cost frontier analysis of network processes under convex technology

Under convex meta cost frontier, the meta cost efficiency of network processes can be obtained using the following model:

$$E_N^{*M-convex} = Min \frac{\sum_{i\in I_1} \overline{x}_i^{meta1} + \sum_{i\in I_2} \overline{x}_i^{meta2}}{\sum_{i\in I_1} c_i x_{io}^1 + \sum_{i\in I_2} c_i x_{io}^2}$$

$$\begin{aligned}
s.t. \quad & \sum_{n=1}^{N'} \sum_{g=1}^{g^n} \lambda_{gn} \overline{x}_{ig}^1 \le \overline{x}_i^{meta1}, \forall i \in I_1, \\
& \sum_{n=1}^{N'} \sum_{g=1}^{g^n} \lambda_{gn} y_{rg}^1 \ge y_{ro}^1, \forall r \in R_1, \\
& \sum_{n=1}^{N'} \sum_{g=1}^{g^n} \lambda_{gn} b_{ug}^1 = b_{uo}^1, \forall u \in U_1, \\
& \sum_{n=1}^{N'} \sum_{g=1}^{g^n} \lambda_{gn} z_{kg} = z_{ko}, \forall k \in K, \\
& \sum_{n=1}^{N'} \sum_{g=1}^{g^n} \mu_{gn} z_{kg} = z_{ko}, \forall k \in K, \\
& \sum_{n=1}^{N'} \sum_{g=1}^{g^n} \mu_{gn} \overline{x}_{ig}^2 \le \overline{x}_i^{meta2}, \forall i \in I_2, \\
& \sum_{n=1}^{N'} \sum_{g=1}^{g^n} \mu_{gn} y_{rg}^2 \ge y_{ro}^2, \forall r \in R_2, \\
& \sum_{n=1}^{N'} \sum_{g=1}^{g^n} \mu_{gn} b_{ug}^2 = b_{uo}^2, \forall u \in U_2, \\
& \mu_{gn}, \lambda_{gn} \ge 0, \forall g, n.
\end{aligned} \tag{5}$$

The unit under investigation is called meta cost efficient generally if and only if the optimal value $E_N^{*M-convex} = 1$ in model (5). Meta cost efficiency for stages 1 and 2 can also be computed in the following ways, respectively:

$$E_1^{*M} = \frac{\sum_{i\in I_1} \overline{x}_i^{meta1}}{\sum_{i\in I_1} c_i x_{io}^1} \tag{6}$$

and

$$E_2^{*M} = \frac{\sum_{i\in I_2} \overline{x}_i^{meta2}}{\sum_{i\in I_2} c_i x_{io}^2} \tag{7}$$

In stages 1 and 2, the individual under assessment is said to be meta cost efficient if and only if $E_1^{*M} = 1$ and $E_2^{*M} = 1$, respectively.

In the following subsection, meta frontier cost efficiency is measured under the non-convex technology.

2.2.2 Meta cost frontier analysis of network processes under non-convex technology

To evaluate non-convex metafrontier cost efficiency of two-stage network processes, the study of Afsharian and Podinovski (Afsharian and Podinovski 2018) is followed and the below mixed integer non-linear model is introduced:

$$
\begin{aligned}
E_N^{*M-non-convex} = Min \ & \sum_{n=1}^{N'} \frac{(\sum_{i\in I_1} \bar{x}_i^{n1q} + \sum_{i\in I_2} \bar{x}_i^{n2q})}{\sum_{i\in I_1} c_i x_{io}^{n1q} + \sum_{i\in I_2} c_i x_{io}^{n2q}} \\
s.t. \quad & \beta^n \sum_{g=1}^{g^n} \lambda_{gn} \bar{x}_{ig}^1 \le \bar{x}_i^{n1q}, \forall i \in I_1, \forall n, \\
& \beta^n \sum_{g=1}^{g^n} \lambda_{gn} y_{rg}^1 \ge \beta^n y_{ro}^{1q}, \forall r \in R_1, \forall n, \\
& \beta^n \sum_{g=1}^{g^n} \lambda_{gn} b_{ug}^1 = \beta^n b_{uo}^{1q}, \forall u \in U_1, \forall n, \\
& \beta^n \sum_{g=1}^{g^n} \lambda_{gn} z_{kg} = \beta^n z_{ko}^q, \forall k \in K, \forall n, \\
& \beta^n \sum_{g=1}^{g^n} \mu_{gn} z_{kg} = \beta^n z_{ko}^q, \forall k \in K, \forall n, \\
& \beta^n \sum_{g=1}^{g^n} \mu_{gn} \bar{x}_{ig}^2 \le \bar{x}_i^{n2q}, \forall i \in I_2, \forall n, \\
& \beta^n \sum_{g=1}^{g^n} \mu_{gn} y_{rg}^2 \ge \beta^n y_{ro}^{2q}, \forall r \in R_2, \forall n, \\
& \beta^n \sum_{g=1}^{g^n} \mu_{gn} b_{ug}^2 = \beta^n b_{uo}^{2q}, \forall u \in U_2, \forall n, \\
& \sum_{n=1}^{N'} \beta^n = 1, \\
& \mu_{gn}, \lambda_{gn} \ge 0, \forall g, n, \beta^n \in \{0,1\}.
\end{aligned}
\tag{8}
$$

$(x_{io}^{1q}, y_{ro}^{1q}, b_{uo}^{1q}, z_{ko}^q, x_{io}^{2q}, y_{ro}^{2q}, b_{uo}^{2q})$ show the components of the unit under evaluation that belongs to technology $q, q \in \Upsilon$. As observed and mentioned, model (8) is non-linear. However, model (8) can be transformed into a linear program by using the changes of variable $\beta^n \lambda_{gn} = \lambda'_{gn}$ and $\beta^n \mu_{gn} = \mu'_{gn}$. Therefore, we have the following linear problem to measure metafrontier cost efficiency of network processes through the non-convex technology:

$$E_N^{*M-non-convex} = Min \quad \sum_{n=1}^{N'} \frac{(\sum_{i\in I_1} \overline{x}_i^{n1q} + \sum_{i\in I_2} \overline{x}_i^{n2q})}{\sum_{i\in I_1} c_i x_{io}^{n1q} + \sum_{i\in I_2} c_i x_{io}^{n2q}}$$

$$\begin{aligned}
s.t. \quad & \sum_{g=1}^{g^n} \lambda'_{gn} \overline{x}_{ig}^1 \le \overline{x}_i^{n1q}, \forall i \in I_1, \forall j, \forall n, \\
& \sum_{g=1}^{g^n} \lambda'_{gn} y_{rg}^1 \ge \beta^n y_{ro}^{1q}, \forall r \in R_1, \forall n, \\
& \sum_{g=1}^{g^n} \lambda'_{gn} b_{ug}^1 = \beta^n b_{uo}^{1q}, \forall u \in U_1, \forall n, \\
& \sum_{g=1}^{g^n} \lambda'_{gn} z_{kg} = \beta^n z_{ko}^q, \forall k \in K, \forall n, \\
& \sum_{g=1}^{g^n} \mu'_{gn} z_{kg} = \beta^n z_{ko}^q, \forall k \in K, \forall n, \\
& \sum_{g=1}^{g^n} \mu'_{gn} \overline{x}_{ig}^2 \le \overline{x}_i^{n2q}, \forall i \in I_2, \forall n, \\
& \sum_{g=1}^{g^n} \mu'_{gn} y_{rg}^2 \ge \beta^n y_{ro}^{2q}, \forall r \in R_2, \forall n, \\
& \sum_{g=1}^{g^n} \mu'_{gn} b_{ug}^2 = \beta^n b_{uo}^{2q}, \forall u \in U_2, \forall n, \\
& \sum_{n=1}^{N'} \beta^n = 1, \\
& \mu'_{gn}, \lambda'_{gn} \ge 0, \forall g, n, \beta^n \ge 0, \forall n.
\end{aligned} \tag{9}$$

Under non-convex technology, the unit under measurement is overall meta cost efficient if and only if $E_N^{*M-non-convex} = 1$. Also, it is meta cost efficient in stages 1 and 2 if and only if

$E_1^{*M-non-convex} = \sum_{n=1}^{N'} (\sum_{i\in I_1} \bar{x}_i^{n1q} / \sum_{i\in I_1} c_i x_{io}^{n1q}) = 1$ and

$E_2^{*M-non-convex} = \sum_{n=1}^{N'} (\sum_{i\in I_2} \bar{x}_i^{n2q} / \sum_{i\in I_2} c_i x_{io}^{n2q}) = 1$, respectively.

Theorem. The group cost efficiency of the two-stage systems evaluated by model (4) is not less than the meta cost efficiency values calculated using model (5) or model (9).

Proof. Under convex meta technology, the production possibility set of model (5) includes all DMU in all groups while the production possibility set of model (4) consists of DMUs of a special group. It is clear that $E^{*M-convex} \le E^{*g}$. Under non-convex meta technology, the optimal value of model (9) is equal to the minimum value of the group cost efficiencies computed by model (4). Thus, $E^{*M-non-convex} \le E^{*g}$. Notice that these relationships are correct for the general two-stage system and for each stage. ❐

2.3. Cost gap ratio and decomposition of meta cost frontier inefficiency

The cost gap ratio is used to express the difference between the group cost frontier and the meta cost frontier. In other words, it measures the cost saving potential for outputs given under meta cost technology. Cost gap ratio (CGR) is computed by the ratio of the meta cost efficiency to the group cost efficiency. Therefore, CGRs for stages 1 and 2 under convex and non-convex metatechnologies are measured in the following ways:

$$CGR_1 = \frac{E_1^{*M-convex}}{E_1^{*g}}, \tag{10}$$

$$CGR_2 = \frac{E_2^{*M-convex}}{E_2^{*g}}, \tag{11}$$

$$CGR_1 = \frac{E_1^{*M-non-convex}}{E_1^{*g}}, \tag{12}$$

and

$$CGR_2 = \frac{E_2^{*M-non-convex}}{E_2^{*g}}. \tag{13}$$

The amount of CGR is defined between zero and one, that is $0 < CGR \le 1$ due to this matter that $E^{*M} \le E^{*g}$. The higher value CGR, the closer is the group cost frontier to the meta cost frontier.

The source of meta cost inefficiency is also decomposed into managerial group cost inefficiency and cost gap inefficiency. Managerial group cost inefficiencies for stages 1 and 2 can be calculated as follows:

$$IE_1^g = 1 - E_1^{*g}, \tag{14}$$

and

$$IE_2^g = 1 - E_2^{*g}. \tag{15}$$

Also, cost gap inefficiency (CGI) scores for stages 1 and 2 can be defined in the following ways, respectively:

$$CGI1 = E_1^{*g} - E_1^{*M} = E_1^{*g}(1 - CGR_1), \tag{16}$$

and

$$CGI2 = E_2^{*g} - E_2^{*M} = E_2^{*g}(1 - CGR_2). \tag{17}$$

Thus, the meta cost inefficiency (MCI) index for the first stage $(MCI1)$ and for the second stage $(MCI2)$ can be delineated as follows:

$$MCI1 = IE_1^g + CGI1, \tag{18}$$

and

$$MCI2 = IE_2^g + CGI2. \tag{19}$$

3. The application

In this section, an example of soft drinks companies is provided to explain the proposed approaches and to show their applicability. Data are partially derived from (Mirhedayatian et al. 2014). We consider each process as a network with two components, supplier and producer. Input (I), desirable output (DO) and undesirable output (UO) measures are as follows and the data set is presented in Table 1.

<u>Performance measures of stage 1</u>

Inputs: Material cost $(I1)$, transportation cost $(I2)$, staff cost $(I3)$, quality cost $(I4)$, advertisement cost $(I5)$ and reliability cost $(I6)$.

Desirable outputs: Facility technology level $(DO1)$, supplier flexibility $(DO2)$, capability of suppliers $(DO3)$ and services $(DO4)$.

Undesirable outputs: Parts per million $(UO1)$.

<u>Performance measures of stage 2</u>

Inputs: Transportation cost $(I1)$ and eco-design cost $(I2)$

Desirable outputs: Producer reputation $(DO1)$ and number of green products $(DO2)$

Undesirable outputs: CO_2 emission $(UO1)$

Intermediate measure: Number of parts from supplier to producer (Z).

Prices for inputs of stages 1 and 2 are equal to one. Companies are divided into two groups due to the approach represented in (Ding et al. 2018). The first group (A) includes Behnoush, Zam Zam, Damdaran, Pegah and Varna. Also, the second group (B) covers Abali, Kafi, Khazar, Sara and Ramak. To assess the group cost efficiency for two groups, model (4) is computed. The results can be found in Table 2. Three companies, Abali, Zam Zam and Khazar are obtained as inefficient in stage 2 while all companies are determined as efficient in stage 1. Also, three companies, Abali, Zam Zam and Khazar are generally group cost inefficient. Afterwards, to measure the meta cost efficiency of companies under convex and non-convex meta technologies, models (5) and (9) are estimated, respectively. The findings are revealed in column 5-10 of Table 2. As can be observed, the group cost efficiencies are obtained equal to the meta cost efficiencies under non-convex technology in this case. Also, the meta cost efficiency scores under convex technology are less than or equal to the group cost efficiency values. Under meta convex technology, only one company that is Kafir is specified as inefficient in stage 1 whilst this amount reach five companies for stage 2. In other words, Abali, Zam Zam, Khazar, Sara and Ramak companies are inefficient in stage 2 under meta convex cost technology. Furthermore, six companies, Abali, Kafir, Zam Zam, Khazar, Sara and Ramak are cost overall inefficient under the meta convex technology. As can be seen in Table 2, more companies are identified as inefficient under the meta convex technology in comparison with the mata non-convex technology in this case. Number of group cost efficient companies and meta cost efficient companies under convex technology for stages suppliers and producers are provided in Figures 2 and 3.

As can be seen, the number of meta cost efficient companies is not more than group cost efficient companies in both groups, A and B. Especially, it can be seen for group B in both stages. Moreover, the number of meta cost efficient companies and group cost efficient companies is equal for group A in supplier and producer stages in this case study.

Table 1 – Data set

Company	Stage1										
	$I1$	$I2$	$I3$	$I4$	$I5$	$I6$	$DO1$	$DO2$	$DO3$	$DO4$	$UO1$
Behnoush	290	220	85	75	104	60	3	2	1250	4	39
Abali	300	345	95	110	125	65	2	2	1295	2	34
Kafir	288	350	110	85	110	72	3	3	1320	3	46
Zam Zam	320	330	80	65	105	78	2	3	1259	3	32
Khazar	290	275	92	93	135	90	4	2	1320	2	53

Damdaran	340	210	103	115	142	88	3	4	1349	2	62
Sara	325	370	100	125	159	92	4	2	1329	4	39
Ramak	330	250	87	150	130	95	2	4	1276	2	45
Pegah	349	320	75	145	115	105	4	3	1293	3	72
Varna	295	335	92	80	100	70	3	3	1302	4	42

Company	Stage 2					Intermediate
	$I1$	$I2$	$DO1$	$DO2$	$UO1$	Z
Behnoush	139	394	3	490	155	236
Abali	125	452	2	523	167	279
Kafir	155	329	3	539	153	247
Zam Zam	132	442	3	597	180	289
Khazar	149	526	2	479	167	275
Damdaran	176	349	3	623	156	298
Sara	125	527	3	589	178	320
Ramak	192	397	2	532	182	327
Pegah	156	309	3	508	167	297
Varna	145	403	3	639	174	217

Table 2 – Group and meta cost efficiencies

Company	Group cost efficiency			Meta convex cost efficiency			Meta non-convex cost efficiency		
	Overall	Stage 1	Stage 2	Overall	Stage 1	Stage 2	Overall	Stage 1	Stage 2
Behnoush	1	1	1	1	1	1	1	1	1
Abali	0.9722	1	0.9221	0.9384	1	0.8273	0.9722	1	0.9221
Kafir	1	1	1	0.9616	0.9432	1	1	1	1
Zam Zam	0.9718	1	0.9236	0.9718	1	0.9236	0.9718	1	0.9236
Khazar	0.9124	1	0.7858	0.8819	1	0.7113	0.9124	1	0.7858
Damdaran	1	1	1	1	1	1	1	1	1
Sara	1	1	1	0.9301	1	0.8045	1	1	1
Ramak	1	1	1	0.9586	1	0.8853	1	1	1
Pegah	1	1	1	1	1	1	1	1	1
Varna	1	1	1	1	1	1	1	1	1

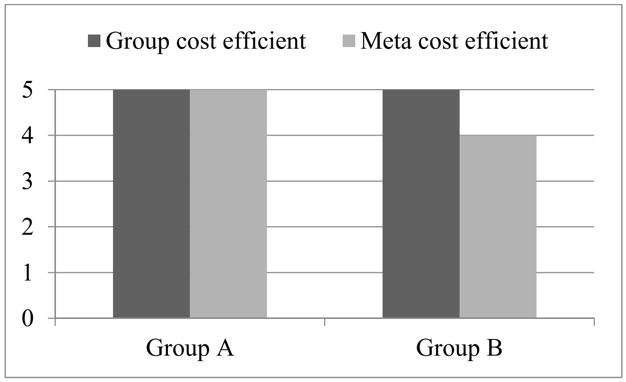

Figure 2 – Number of cost efficient companies in the supplier stage

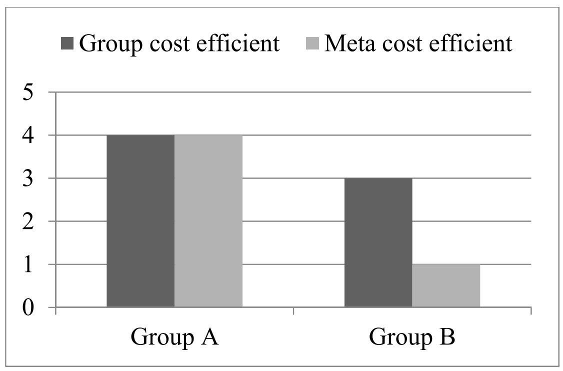

Figure 3 – Number of cost efficient companies in the producer stage

Also, the cost gap ratios for stages 1 and 2 are estimated by using expressions (10) and (11) under convex technology and by applying statements (12) and (13) under non-convex technology. The results are found in Table 3. As can be seen, the average cost gap ratios under meta convex technology for stages 1 and 2 are equal to 0.9943 and 0.9492, respectively. It is clear that it is more in stage 1 in comparison to that in stage 2. Under non-convex technology, the cost gap ratio is equal to one for both stages. Higher value CGR shows the group cost frontier is closer to the meta cost frontier. Under non-convex technology, the average cost gap ratios for groups A and B in stages 1 and 2 are obtained equal to one. It means that there is no gap between group cost frontier and meta cost frontier in these cases. Also, this condition is observed under convex technology and for group A in stages 1 and 2. Actually, the difference between group cost efficiency and meta cost efficiency is not detected. Under convex technology, cost gap ratios for group B in stages 1 and 2 are 0.9886 and 0.8984, respectively. To illustrate, the average cost gap ratios for group B are less than group A in both stages. This implies that the potential of the cost efficiency improvement for group B is more than group A in both stages.

Table 3 – Cost gap ratios

Company	Convex Technology		Non-convex Technology	
	CGR1	CGR2	CGR1	CGR2
Behnoush	1	1	1	1
Abali	1	0.8972	1	1
Kafir	0.9432	1	1	1
Zam Zam	1	1	1	1
Khazar	1	0.9052	1	1
Damdaran	1	1	1	1
Sara	1	0.8045	1	1
Ramak	1	0.8853	1	1
Pegah	1	1	1	1
Varna	1	1	1	1
Mean	0.9943	0.9492	1	1
	Mean		Mean	
	CGR1	CGR2	CGR1	CGR2
Group A	1	1	1	1
Group B	0.9886	0.8984	1	1

Table 4 indicates the findings of group cost inefficiency and meta cost inefficiency for stages 1 and 2. Lower values of group cost inefficiency and meta cost inefficiency show better performance within the group and all companies. The more CGI score, the farther distance between group cost frontier and meta cost frontier. Under non-convex meta technology, CGI for stages 1 and 2 are obtained equal to zero that implies there is no cost gap. But, under convex technology, CGI1 for Kafir is equal to 0.0568 while for other companies CGI1 is equal to zero, meaning that there is the cost gap for Kafir in stage 1. In stage 2, there are both managerial group cost inefficiency and cost gap inefficiency in some companies such as Abali and Khazar.

Also, the average MCI1 and MCI2 of group A are less than group B under convex meta technology that shows better performance of group A compared to group B. MCI1 is found equal to zero for group A under convex meta technology and for groups A and B under non-convex meta technology which means they have the

best performance. As evidenced from Table 4, the average MCI2 for group A is less than group B under non-convex meta technology stating that group A operates better than group B in stage 2.
Generally, meta inefficiency is not detected for group A in stage 1 under convex meta technology and for groups A and B in stage 1 under non-convex meta technology.

Table 4 – Decomposition of cost inefficiencies

Company			Convex meta technology				Non-convex meta technology			
	IE1	IE2	CGI1	CGI2	MCI1	MCI2	CGI1	CGI2	MCI1	MCI2
Behnoush	0	0	0	0	0	0	0	0	0	0
Abali	0	0.0779	0	0.0948	0	0.1727	0	0	0	0.0779
Kafir	0	0	0.0568	0	0.0568	0	0	0	0	0
Zam Zam	0	0.0764	0	0	0	0.0764	0	0	0	0.0764
Khazar	0	0.2142	0	0.0745	0	0.2887	0	0	0	0.2142
Damdaran	0	0	0	0	0	0	0	0	0	0
Sara	0	0	0	0.1955	0	0.1955	0	0	0	0
Ramak	0	0	0	0.1147	0	0.1147	0	0	0	0
Pegah	0	0	0	0	0	0	0	0	0	0
Varna	0	0	0	0	0	0	0	0	0	0
	Mean						Mean			
	IE1	IE2	CGI1	CGI2	MCI1	MCI2	CGI1	CGI2	MCI1	MCI2
Group A	0	0.0153	0	0	0	0.0153	0	0	0	0.0153
Group B	0	0.0584	0.0114	0.0959	0.0114	0.1543	0	0	0	0.0584

4. Conclusions

In this paper, an approach based on DEA was proposed to analyze the cost efficiency of heterogeneous two-stage processes with undesirable outputs. The weak disposability assumption has been considered for undesirable outputs and approaches were based on constant returns to scale property. However, the introduced approach can also be extended to estimate the cost efficiency of nonhomogeneous two-stage systems under the variable returns to scale assumption. Furthermore, cost gap ratios and resources of meta cost frontier inefficiency were assessed. To illustrate in more details, they were calculated for each stage of network processes. Due to presence of convex and non-convex metasets in many applications, the meta cost efficiency has been evaluated under both convex and non-convex technologies. The suggested technique has been explained with an example from the literature that indicates the applicability and suitability the provide approach to investigate group cost efficiency, meta convex cost efficiency and meta non-convex cost efficiency of two-stage systems with undesirable outputs.

Despite the fact that the introduced approach in this paper is beneficial, but in the broader sense, the analysis of the cost performance of heterogeneous two-stage systems with imprecise data is an interesting topic to examine. Also, the study can be performed to measure the cost efficiency of heterogeneous network systems in the presence of forward and backwards flows.

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